

Isolating the vortex core Majorana state in p-wave superconductors.

B. Rosenstein^{1,2}, I. Shapiro³, B. Ya. Shapiro³

¹*Department of Electrophysics, National Chiao*

Tung University, Hsinchu, Taiwan, R.O.C. and

²*Applied Physics Department, Ariel University Center of Samaria, Ariel 40700, Israel*

³*Department of Physics, Institute of Superconductivity,*

Bar-Ilan University, Ramat-Gan 52900, Israel

Abstract

The spectrum of core excitations of the Abrikosov vortex pinned by a nanohole of the size of the coherence length is considered. While the neutral zero energy Majorana core state remains intact due to its topological origin, the energy of charged excitations is enhanced significantly compared to that in the unpinned vortex. As a consequence of the pinning the minigap separating the Majorana state from the charged levels increases from Δ^2/E_F for (E_F is Fermi energy, Δ - the bulk p -wave superconducting gap) to a significant fraction of Δ . Suppression of the thermodynamic and kinetic effects of the charged excitations allows to isolate the Majorana state so it can be used for quantum computation. It is proposed that thermal conductivity along the vortex cores is a sensitive method to demonstrate the minigap. We calculate, using Butticker - Landauer - Kopnin formula, the thermal conductance beyond linear response as function of the hole radius.

Spin-triplet p -wave superfluids, both neutral, such as liquid He^3 (and recently generated by the Feshbach resonance on Li^6 and K^{40}) and charged such as superconducting material Sr_2RuO_4 and possibly heavy fermion UPt_3 have resulted in very rich physics [1]. The condensate is described by a generally tensorial complex order parameter Δ exhibiting great variety of the broken symmetries ground states. The broken symmetry and boundary conditions give rise to the continuous configuration of the order parameter as nontrivial topological excitations [2]. Especially interesting is the case of the so-called topological superconductors, characterized by presence of electron-hole symmetry and absence of both the time-reversal and spin-rotation symmetry. Realizations of topological p -wave superfluids are chiral superconductors like Sr_2RuO_4 , with order parameter of the $p_x \pm ip_y$ symmetry type [3] and the ABM - phase[1] of superfluid He^3 and other fermionic cold atoms [4] and topological superconductor $Cu_xBi_2Se_3$ that produces an equivalent pseudospin system on its surface[5].

Magnetic field in type II superconductors easily creates stable line - like topological defects, Abrikosov vortices[6], see Fig.1a. In the simplest vortex the phase of the order parameter rotates by 2π around the vortex and each vortex carries a unit of magnetic flux Φ_0 . Quasiparticles near the vortex core "feel" the phase wind by creating a set of discrete low-energy Andreev bound state. For the s -wave superconductors when the vortices are unpinned (freely moving) these states were comprehensively studied theoretically including the excitations spectrum [7], density of states [8], their role in vortex viscosity[6], contribution to the heat transport[9] and to the microwave absorption [10]. The low lying spectrum of quasiparticle and hole excitations is equidistant, $E_l = l\omega$, where angular momentum l takes half integer values. The "minigap" in the s -wave superfluids is of order of $\omega = \Delta^2/E_F \ll \Delta$, where Δ is the energy gap and E_F is the Fermi energy.

Free vortices in the p -wave superconductors exhibit a remarkable topological feature of appearance of the zero energy mode in the vortex core [11]. The spectrum of the low energy excitations remains equidistant, $E_l = (l - 1)\omega$, but now l is integer[12]. The zero mode represents a condensed matter analog of the Majorana fermion first noticed in elementary particle physics[13]. Its remarkable feature is linked to the fact that its creation operator is identical to its own annihilation, called the state. Its topological nature ensures robustness against perturbations from deformations of order parameters and nonmagnetic impurities. The states obey non-Abelian statistics with their pairs constituting a qubit. This might

offer a promising method of the fault-tolerant quantum computation [16]. The main issue is to isolate the states from those above the minigap [14].

The $l = 1$ Majorana mode in the p - wave vortex states has been a popular topic of study over the last several years [12, 15, 17, 18]. While the minigap in the s and d -wave superconductors was detected by STM [21], in p -wave it has not been observed. The major reason for that is the *small value of the minigap* ω in the core spectrum (just mK for Sr_2RuO_4). To address this problem a current trend was to propose increasingly sophisticated combinations of materials and geometries. For example one of the proposals [20] to expose the Majorana state is to induce the s -wave superconductivity by the proximity effect on the surface of a topological insulator. The minigaps of the resultant non-Abelian states can be orders of magnitude larger than in a bulk chiral p -wave superconductor.

In this note we propose to solve the Majorana minigap problem within the original system, a $p_x + ip_y$ bulk superconductor in magnetic field by pinning the vortices on artificially fabricated dielectric inclusions of the radius comparable to the coherence length ξ of the superconductor [22]. It was shown theoretically [24] that in the s -wave superconductors pinning by an inclusion of radius of just $R = 0.2 - 0.5\xi$ changes dramatically the subgap excitation spectrum: the minigap $\omega \sim \Delta^2/E_F$, becomes of the order of Δ . In the present note we present the spectrum and wave functions of the core excitations in the chiral p -wave superconductor. The charged states for $R = 0.1 - 0.4\xi$ are significantly pushed up towards Δ , so that they therefore interfere less with Majorana state. To expose the modified charged spectrum we propose to measure heat transport along the vortex axis, see Fig.1b. The temperature gradient between the top side and the bottom side drives heat along the vortex axis[9]. The exponential temperature dependence of thermal conductivity is very sensitive to the minigap. The temperature when the material undergoes insulator - thermal conductor crossover rises from $\Delta/100$ for unpinned vortices to $\Delta/10$ for inclusion radius $R = 0.4\xi$.

We start with the Bogoliubov - de Gennes (BdG) equations for the $p_x + ip_y$ superconductor in the presence of a single pinned vortex. The vector potential \mathbf{A} in polar coordinates, r, φ , has only an azimuthal component $A_\varphi(r)$ and in the London gauge consists of the singular part $A_\varphi^s = hc/2er$ and the regular part of the vector potential that can be neglected for a type II superconductor [12]. In the operator matrix form for a two component amplitude the BdG equations read:

$$\begin{pmatrix} \hat{H}_0 & L \\ L^+ & -\hat{H}_0^* \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = E \begin{pmatrix} u \\ v \end{pmatrix}, \quad (1)$$

where for anisotropic dispersion

$$\begin{aligned} H_0 &= -\frac{\hbar^2}{2m_\perp} \nabla_\perp^2 - \frac{\hbar^2}{2m_z} \nabla_z^2 - E_F; \\ L &= -\frac{\Delta}{k_F} \left\{ s(r) e^{i\varphi} (i\nabla_x - \nabla_y) + \frac{1}{2} [(i\nabla_x - \nabla_y) s(r) e^{i\varphi}] \right\}, \end{aligned} \quad (2)$$

with Δ being the "bulk gap" of order T_c . The equations, possess the electron-hole symmetry. The Ansatz

$$\begin{aligned} u &= \frac{1+i}{\sqrt{2}} f(r) e^{il\varphi} e^{ik_z z} \\ v &= \frac{1-i}{\sqrt{2}} g(r) e^{i(l-2)\varphi} e^{ik_z z} \end{aligned} \quad (3)$$

converts them (for any l there are radial excitation levels denoted by n) into a dimensionless form,

$$\begin{aligned} -\gamma \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{l^2}{r^2} + \frac{1}{4\gamma^2} \right) f - 2\gamma \left[s(r) \left(\frac{\partial}{\partial r} - \frac{l-2}{r} \right) + \frac{1}{2} \left(s'(r) - \frac{s(r)}{\bar{r}} \right) \right] g &= \varepsilon_{lk_z n} f, \\ \gamma \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{(l-2)^2}{r^2} + \frac{1}{4\gamma^2} \right) g + 2\gamma \left[s(r) \left(\frac{\partial}{\partial r} + \frac{l}{r} \right) + \frac{1}{2} \left(s'(r) - \frac{s(r)}{\bar{r}} \right) \right] f &= \varepsilon_{lk_z n} g, \end{aligned}$$

with dimensionless energy $\varepsilon_{lk_z n} = E_{lk_z n}/\Delta$. Here distances are in units of ξ . We chose the order parameter as a discontinuous function vanishing inside the core $r < R$ and $s = \tanh(r)$ for $r > R$. (The question of justification of using this form often used instead of the fully self consistent approach was extensively studied in literature on both s-wave and non-conventional pairing [8], [12]). In the clean limit BCS (applicable to $SrRu_2O_4$) $\xi = \hbar k_\perp / m_\perp \Delta$, where

$$k_\perp^2 / 2m_\perp = E_F / \hbar^2 - k_z^2 / 2m_z, \quad (5)$$

and for given k_z there is just one dimensionless parameter

$$\gamma = 1/2k_\perp \xi = m_\perp \Delta / 2\hbar^2 k_\perp^2. \quad (6)$$

The Ansatz, Eq.(3) was chosen in such a way that the equations become real. In the presence of a hole of radius R we assume that the order parameter profile is still accurate for $r > R$. In a microscopic theory of the superconductor-insulator interface, (see [23]), the

order parameter rises abruptly from zero in dielectric, where amplitudes of normal excitations $f = g = 0$, to a finite value inside the superconductor within an atomic distance a from the interface, namely with a slope $\propto 1/a$. This means that the boundary condition on the amplitudes is consistent with zero order parameter at the boundary point $r = R - a$ in the self consistency equation. The sample will be cylindric with radius L , see Fig.1a.

Qualitatively the spectrum of a single vortex in a hollow disk with internal and external radii R and L consists of two Majorana states, several pairs of Andreev bound subgap states both quasiparticles and holes (related to each other by the electron - hole symmetry) and a continuum of states above a threshold at T_c . The spectrum was calculated numerically by using NAG Fortran Library Routine Document F02EBF. It computes all the eigenvalues, and optionally all the eigenvectors, of a real general matrix, for various values of the inclusion radius $R = 0 \sim 0.9\xi$, external radius $L \sim 10 - 20$ and the universal parameter $\gamma(k_z) = 1/2k_\perp\xi \sim 10^{-3} - 3 \cdot 10^{-2}$. For sufficiently small l , in addition to the continuum of states above the superconducting gap with $n \geq 1$, there are bound Andreev states that correspond to the lowest $n \equiv 0$. For $l = 1$ and various inclusion radii R there is the Majorana state, for which $f(r) = g(r)$ near the "internal" surface" extending to the distance ξ into the superconductor, see Fig.2a where the radial density,

$$\rho(r) = 2\pi r (|f(r)|^2 + |g(r)|^2), \quad (7)$$

is given for sufficiently large external radius of cylinder, $L = 20$ and $\gamma = 0.03$. As was noticed in ref. [17], the function oscillates with period $1/k_\perp\xi$. There is also the second Majorana mode on the "external" surface for which $f(r) = -g(r)$, given in Fig.2b. The energy of the both states is exponentially small $\varepsilon_{1k_z} \propto e^{-L}$ due to tunneling between them[19]. The surface state wave function practically does not depend on the inclusion radius for all values of R considered, although it does depend on L . When one considers a small cylinder of the width of several coherence length like the one shown in Fig.1a, the two Majorana modes start to overlap. All the other $l = 1$ excitations, $n \geq 1$, are above the threshold and will not be considered and the index omitted.

At angular momenta $l \neq 1$ topology does not protect the energy (more precisely its absolute value). The wave function of bound states, is pushed out of its position near the vortex core when the inclusion is absent. The state that was localized low energy at $R = 0$, becomes delocalized and approaches the threshold at $R = 0.2\xi$ and eventually at $R = 0.4\xi$

merges into the threshold. The energies in units of Δ for wide range of angular momenta l for $R = 0$ (the blue points), 0.1ξ (green), 0.2ξ (red), 0.4ξ (brown) are given in Fig. 3. The $R = 0$ line is well approximated for $|l| < 10$ by the semiclassical linear formula [11], while for higher momenta it approaches the threshold along a universal curve that is independent of the inclusion radius. When the hole is present, the dependence on l is no longer monotonic. It first rises to a maximum at $l = -1$ and subsequently has a local minimum with energy about that of the $l = 0$ state. Consequently this becomes a new minigap that becomes of order of the bulk gap already at relatively small inclusion radius. Beyond this minimum the curve approaches the universal $R = 0$ spectrum, so that wave functions are located far from the inclusion. Beyond radius $R = 0.4\xi$ the effect of the minigap enhancement is saturated. Only the value of $\gamma = 0.03$ is shown in Fig.3 although dependence on it is very weak for all $\gamma \ll 1$. The enhanced minigap allows to utilize the Majorana states.

Thermal conductivity is an effective tool to demonstrate the minigap due to activated behavior of the electron contribution. To calculate the quasiparticle contribution to thermal conductivity along the vortex cores when the upper side of the vortex line is held at temperature T_1 and the lower side at temperature T_2 , see Fig.1b, we use a general ballistic (width of the film L_z smaller than mean free path) Kopnin-Landauer formula[9]. The heat current at temperature lower than the threshold to continuum of states is carried by the bound core states (except the Majorana). For a single vortex it consists of the contribution of quasiparticles and holes, $I = 2 \sum_{l < 1} I_l(T_2) - I_l(T_1)$,

$$I_l(T) = \int_0^{k_z^{\max}} \frac{dk_z}{2\pi\hbar} \left| \frac{dE_{lk_z}}{dk_z} \right| \frac{E_{lk_z}}{1 + \exp(E_{lk_z}/T)}, \quad (8)$$

where the energy depends on transferred momentum along the field k_z via γ , see Eq.(6). The maximal value of k_z is $k_z^{\max} = \sqrt{2m_z E_F}$. Since E_{lk_z} is monotonic the integral can be transformed into

$$I_l(T) = \int_{E_l^0}^{E_F} \frac{dE}{2\pi\hbar} \frac{E}{1 + \exp(E/T)} \approx \frac{T^2}{2\pi\hbar} \Pi\left(\frac{E_{l0}}{T}\right). \quad (9)$$

Since the temperatures are below the threshold, E_F was replaced in the upper limit of the integral by ∞ . Here lower limit of integration is energy for $k_z = 0$ and

$$\Pi(x) = \pi^2/6 - x^2/2 + x \log(1 + e^x) + Li_2(-e^x), \quad (10)$$

where Li is the polylog function. For small temperature differences the linear response can be used,

$$\frac{\hbar}{\Delta} \frac{dI}{dT} = \frac{t}{\pi} \sum_{\iota < 1} \left[2\Pi \left(\frac{\varepsilon_{l0}}{t} \right) + \frac{(\varepsilon_{l0}/t)^2}{1 + \exp(\varepsilon_{l0}/t)} \right], \quad (11)$$

where $t = T/\Delta$. The values of energies of the core states, presented in Fig. 3, therefore allow to calculate the thermal conductivity. In Fig.4 the heat conductance of a single vortex line is given as function of inverse temperature (in units of Δ^{-1}). While for unpinned vortex (the blue line) the crossover from thermal insulator to conductor is not well defined in the relevant temperature range $\Delta/100 < T < \Delta/4$, for the radius of the inclusion $R = 0.1\xi, 0.2\xi, 0.4\xi$ (brown, red and green respectively) one observes a well defined crossover temperature.

To summarize, the wave functions and the spectrum of the core states well pinned vortices in the chiral p wave superconductor in magnetic field was studied, see Fig. 3. The pinning by a nanohole of the order of coherence ξ is assumed. It is shown that while the neutral Majorana mode is largely intact, all the other Andreev bound states have their energies significantly lifted. The minigap (the energy difference with the Majorana state) is consequently increased from its value for unpinned vortices, Δ^2/E_F , to a fraction of the bulk superconducting gap Δ . In superconductor Sr_2RuO_4 while the minigap for unpinned vortices is just (using [3] $\Delta = 2K$, $E_F = 10^3 K$) $\omega = 4mK$, it would become about $0.2K$ for a nanohole of radius of ($\xi = 65nm$ [3]) $R = 0.4\xi = 25nm$. This would suppress the thermodynamic and transport effects of the charged excitations and allow to observe the physics of the Majorana state.

We propose to demonstrate the enhanced minigap by measuring thermal conductivity along the vortex direction. The activated behavior of the electron contribution per vortex is given in Fig.4 and compared to the phonon contribution that dominates at temperatures below $T_m = \Delta/10$. Magnetic field $B \gg H_{c1}$ creates $N = SB/\Phi_0$ vortices, see Fig. 1b, over area S , so that heat conductivity is $\kappa = \frac{L_z B}{\Phi_0} \frac{dI}{dT}$, where L_z is the sample width. For $B = 0.5T$ (between H_{c1} and H_{c2} for in Sr_2RuO_4), with $R = 25nm$; $L_z = 70nm$ at $T = 0.2K$ one obtains conductivity $\kappa = 0.05 W/Km$. The phonon contribution in ballistic regime ($L_z \ll l_{ph}$, where l_{ph} is the phonon mean path) is [25] $\kappa \propto C v_{ph} L_z$. Taking the phonon heat capacity C in Debye approximation, one obtains for phonon conductance per vortex,

$$\hbar \kappa_{ph} / \Delta = t^3 (\Delta / \Theta_D)^3 L_z n_A \Phi_0 / B, \quad (12)$$

where Θ_D , and n_A are the Debye temperature, density of the atoms. This dependence for various vortex densities presented in Fig.4 (dashed lines) demonstrates that for temperature above the minigap value, the quasiparticle heat conductance clearly dominates. In particular for parameters $L_z = 50nm$, $B = 0.5T$, $a = 0.5nm$, $T_m = 0.2$ (here a is the inter-atomic distance).

It should be noted that vortices in such an experiment (for the field cooling protocol) will be trapped by the holes rather than by point defects. Those initially not pinned by the holes can be effectively pushed out by a small bias current. Our experimental proposal does not require an ideal hexagonal lattice of identical holes. Periodicity does not play a role and the array may even be random.

Acknowledgements. B.Ya.S. and I.S. acknowledge support from the Israel Scientific Foundation.

Figure captions

Fig.1a.

A single vortex in type II superconductor in magnetic field pinned on an insulator insertion of radius $R \sim \xi$ parallel to the field. The radius of the superconducting disk is $L \gg \xi$. The geometry used to calculate the spectrum of the chiral p -wave core states.

Fig. 1b.

Heat flow through a superconductor in magnetic field along the vortex cores. The temperature difference between the bottom (T_2) and the top (T_1) contacts leads to energy flow carried by both the neutral (Majorana) and charged core states. Numerous vortices are pinned in inclusions and might be arranged as a lattice.

Fig.2a.

The radial density $\rho(r)$, Eq.(7), of the Majorana core mode localized on the inclusion (green area) of radius $R = 0.2\xi$ as function of the distance from the vortex center. The value of the only parameter characterizing the system is $\gamma = 0.03$. The energy of the state exponentially small as function of the sample radius L . The wave function is pushed out of the center by the inclusion, but otherwise remains intact compared to the unpinned vortex. It extends several coherence lengths inside the superconductor.

Fig.2b.

The radial density $\rho(r)$ as function of the distance from the vortex center, Eq.(7), of the Majorana mode localized on the surface the sample at $r = L = 20\xi$. Due to tunneling to the Majorana core state it also has a negligible energy.

Fig. 3.

Excitation energy (in units of the bulk gap Δ) of the bound states as function of angular momentum for several values of the inclusion radius R at $\gamma = 0.03$. For unpinned vortices $R = 0$ (the blue points) the spectrum at small l is described well by the semiclassical Volovik formula (see [11]), while for higher momenta it gradually approaches the threshold along a universal curve that is independent of the inclusion radius R . When the inclusion is present the dependence on l is no longer monotonic. It first rises to a maximum at $l = -1$ and subsequently has a local minimum with energy about that of the $l = 0$ state.

Fig.4.

Dimensionless heat conductance $\frac{\hbar}{\Delta} \frac{dI}{dT}$ of a single vortex given in Eq.(11) as function

of inverse temperature in units of Δ^{-1} . The black line is for unpinned vortex, while the corresponding crossover temperature from thermal insulator to conductor for pinned vortices on insulating inclusions of radius $R = 0.1\xi$ (brown), $R = 0.2\xi$ (red), $R = 0.4\xi$ (green) are much higher. Dashed lines are the contributions of phonons per vortex for $B = 0.5T$ (green), $0.2T$ (black), $0.1T$ (blue), length $L_z = 50nm$, density $n_A = 10^{23}cm^{-3}$, $\hbar\omega_D = 400K$.

-
- [1] A. J. Leggett, Rev. Mod. Phys. **47**, 331 (1975); A.J. Legget, "Quantum Liquid: Bose Condensation and Cooper pairing in Condensed-Matter Systems", Oxford University Press, Oxford (2006).
 - [2] M. M. Salomaa and G. E. Volovik, Rev. Mod. Phys. **59**, 533 (1987); G. E. Volovik, "Universe in a Helium Droplet", Oxford University Press, London (2003).
 - [3] Y. Maeno, H. Hashimoto, K. Yoshida, S. Nishizaki, T. Fujita, J. G. Bednorz, and F. Lichtenberg, Nature **372**, 532 (1994); A. P. Mackenzie and Y. Maeno, Rev. Mod. Phys. **75**, 657 (2003).
 - [4] M. Sato, Y. Takahashi, and S. Fujimoto, Phys. Rev. Lett. **103**, 020401 (2009); I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008).
 - [5] Y. S. Hor et al, Phys. Rev. Lett. **104**, 057001 (2010); L. A. Wray, Nat. Phys. **6**, 855 (2010).
 - [6] N. Kopnin, "Vortices in type-II superconductors: Structure and Dynamics", Oxford University Press, Oxford, (2001); B. Rosenstein and D.P. Li, Rev. Mod. Phys. **82**, 109 (2010).
 - [7] C. Caroli, P.G. de Gennes, and J. Matricon, Phys. Lett., **9**, 307 (1964).
 - [8] J. D. Shore, M. Huang, A.T. Dorsey, and J. P. Sethna, Phys. Rev. Lett. **62**, 3089 (1989); F. Gygi and M. Schlüter, Phys. Rev. **B41**, 822 (1990); Phys. Rev. **B43**, 7609 (1991); C. Berthod, Phys. Rev. **B71**, 134513 (2005).
 - [9] N. B. Kopnin, A. S. Mel'nikov, and V. M. Vinokur, Phys. Rev. **B68**, 054528 (2003).
 - [10] B. Janko and J. D. Shore, Phys. Rev. **B46**, 9270 (1992).
 - [11] G. E. Volovik, JETP Lett. **70**, 609 (1999).
 - [12] M. Matsumoto and M Sigrist, J. Phys. Soc. Jap. **68**, 724, (1999); C.-K. Lu and S.-K. Yip, Phys. Rev. **B78**, 132502 (2008); S. Fujimoto, Phys. Rev. **B77**, 220501(R) (2008); M. Sato and S. Fujimoto, Phys. Rev. **B79**, 094504 (2009).
 - [13] F. Wilczek, Nat. Phys. **5**, 619 (2009).
 - [14] M. Cheng, R. M. Lutchyn, and S. Das Sarma, Phys. Rev. **B 85**, 165124 (2012).
 - [15] K. Machida and H. Nakanishi, Phys. Rev. **B30**, 122 (1984); K. Machida and M. Fujita, Phys. Rev. **B30**, 5284 (1984); M. Stone and R. Roy, Phys. Rev. **B69**, 184511 (2004).
 - [16] A. Kitaev, Ann. Phys. (NY) **303**, 2 (2003); D. A. Ivanov, Phys. Rev. Lett. **86**, 268 (2001); M. Stone and S.-B. Chung, Phys. Rev. **B73**, 014505 (2006); S. Tewari, S. Das Sarma, C. Nayak,

- C. Zhang, and P. Zoller, Phys. Rev. Lett. **98**, 010506 (2007); C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das Sarma, Rev. Mod. Phys. **80**, 1083 (2008).
- [17] M. Cheng, R. M. Lutchyn, V. Galitski, and S. Das Sarma, Phys. Rev. Lett. **103**, 107001(2009).
- [18] V. Gurarie and L. Radzihovsky, Phys. Rev. **B75**, 212509 (2007).
- [19] T. Mizushima and K. Machida, Phys. Rev. **A82**, 023624 (2010).
- [20] J. D. Sau, R. M. Lutchyn, S. Tewari, and S. Das Sarma, Phys. Rev. **B82**, 094522 (2010).
- [21] H. F. Hess, R.B. Robinson, R.C. Dynes, J.M. Valles, J.V. Waszczak, Phys. Rev. Lett. **62**, 214 (1989); L. Shan, Y.-L. Wang, B. Shen, B. Zeng, Y. Huang, A. Li, D. Wang, H. Yang, C. Ren, Q.-H. Wang, S. H. Pan, and H.-H. Wen, Nature Phys. **7**, 325 (2011).
- [22] H. Q. Nguyen, S. M. Hollen, M. D. Stewart, J. Shainline, A. Yin, J. M. Xu, and J. M. Valles, Phys. Rev. Lett. **103**, 157001 (2009); A. Yu. Aladyshkin et al, Supercond. Sci. Technol. **22**, 053001 (2009); V. R. Misko, D. Bothner, M. Kemmler, R. Kleiner, D. Koelle, F. M. Peeters, and F. Nori, Phys. Rev. **B82**, 184512 (2010); J. C. Keay, P. R. Larson, K. L. Hobbs, M. B. Johnson, J. R. Kirtley, O. M. Auslaender, and K. A. Moler, Phys. Rev. **B80**, 165421 (2009); I. Sochnikov, A. Shaulov, Y. Yeshurun, G. Logvenov, and I. Božović, Nature Nano **5**, 516 (2010).
- [23] P. G. DeGennes, "Superconductivity of metals and alloys", W.A. Benjamin inc., New York (1966).
- [24] A. S. Mel'nikov, A. V. Samokhvalov, and M. N. Zubarev, Phys. Rev. **B79**, 134529 (2009); B. Rosenstein, I. Shapiro, E. Deutch, and B.Ya. Shapiro, Phys. Rev. **B84**, 134521 (2011).
- [25] L.D. Landau and E.M. Lifshitz, "Physical kinetics", Pergamon press, Oxfors (1981).

This figure "fig1a.gif" is available in "gif" format from:

<http://arxiv.org/ps/1210.1835v1>

This figure "fig1b.gif" is available in "gif" format from:

<http://arxiv.org/ps/1210.1835v1>

This figure "fig2a.gif" is available in "gif" format from:

<http://arxiv.org/ps/1210.1835v1>

This figure "fig2b.gif" is available in "gif" format from:

<http://arxiv.org/ps/1210.1835v1>

This figure "fig3.gif" is available in "gif" format from:

<http://arxiv.org/ps/1210.1835v1>

This figure "fig4.gif" is available in "gif" format from:

<http://arxiv.org/ps/1210.1835v1>